CP ALGEBRA I & FRESHMAN GEOMETRY

SUMMER PACKET

CONWELL-EGAN CATHOLIC HIGH SCHOOL

This summer assignment is a review and exploration of key skills that are necessary for success in your mathematics course as well as future high school mathematics courses.

All summer packets are due on the Friday of the first full week of school.

Fraction Operations

Hints/Guide:

When adding and subtracting fractions, we need to be sure that each fraction has the same denominator, then add or subtract the numerators together. For example:

$$\frac{1}{8}+\frac{3}{4}=\frac{1}{8}+\frac{6}{8}=\frac{1+6}{8}=\frac{7}{8}$$

That was easy because it was easy to see what the new denominator should be, but what about if it was not so apparent? For example: $\frac{7}{12}+\frac{8}{15}=$

For this example, we must find the Lowest Common Denominator (LCM) for the two denominators 12 and 15.

Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, . . .
Multiples of 15 are 15, 30, 45, 60, 75, 90, 105, . . .
The LCM of 12 and 15 is 60

So, $\frac{7}{12}+\frac{8}{15}=\frac{35}{60}+\frac{32}{60}=\frac{35+32}{60}=\frac{67}{60}=1\frac{7}{60}$.

Note: Be sure that answers are always in lowest terms

To multiply fractions, we multiply the numerators together and denominators together, and then simplify the product. To divide fractions, we find the reciprocal of the second fraction (flip the numerator and the denominator) and then multiply the two together. For example:

$$\frac{2}{3}\bullet\frac{1}{4}=\frac{2}{12}=\frac{1}{6} \text{ and } \frac{2}{3}\div\frac{3}{4}=\frac{2}{3}\bullet\frac{4}{3}=\frac{8}{9}$$

Exercises: Perform the indicated operation No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. $\frac{6}{7}+\frac{2}{3}=$
2. $\frac{8}{9}+\frac{3}{4}=$
3. $\frac{9}{11}-\frac{2}{5}=$
4. $\frac{5}{7}-\frac{5}{9}=$
5. $\frac{6}{11}\bullet\frac{2}{3}=$
6. $\frac{7}{9}\bullet\frac{3}{5}=$
7. $\frac{6}{7}\div\frac{1}{5}=$
8. $\frac{7}{11}\div\frac{3}{5}=$
9. $\left[\frac{2}{3}-\frac{5}{9}\right]\div\left[\frac{4}{7}+\frac{1}{6}\right]=$
10. $\frac{3}{4}+\frac{4}{5}\left[\frac{5}{9}+\frac{9}{11}\right]=$
11. $\left[\frac{3}{4}+\frac{4}{5}\right]\left[\frac{5}{9}+\frac{9}{11}\right]=$

Decimal Operations

Hints/Guide:

When adding and subtracting decimals, the key is to line up the decimals above each other, add zeroes so all of the numbers have the same place value length, then use the same rules as adding and subtracting whole numbers. The answer will have a decimal point in line with the problem.

For example:

$$34.5 + 6.72 + 9.045 = \begin{array}{r} 34.5 \\ 6.72 \\ \underline{9.045} \\ 50.265 \end{array}$$

To multiply decimals, the rules are the same as with multiplying whole numbers, until the product is determined and the decimal point must be located. The decimal point is placed the same number of digits in from the right side of the product as the number of decimal place values in the numbers being multiplied. For example,

8.54 · 17.2, since 854 · 172 is 146888, then we count the number of decimal places in the factors (3) and move in from the right three places, so the final product is 146.888

To divide decimals by a whole number, the division process is the same as for whole numbers, but the decimal points are lined up in the dividend and the quotient. For example, to divide 51.06 by 3, the process is the same as if the problem were 5,106 divided by 3, with the decimal point from the quotient moving up into the quotient to create the final answer of 17.02

$$\begin{array}{r} 17.02 \\ 3\overline{)51.06} \end{array}$$

Exercises: Perform the indicated operation No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. 15.709 + 2.34 + 105.06 =

2. 64.308 + 164.18 + 1005.7 =

3. 87.4 – 56.09 =

4. 500.908 – 4.72 =

5. 6108.09 – 2004.704 =

6. 9055.3 – 242.007 =

7. $\begin{array}{r} 63 \\ \underline{\text{x } .04} \end{array}$

8. $\begin{array}{r} .87 \\ \underline{\text{x } .23} \end{array}$

9. $\begin{array}{r} 8.904 \\ \underline{\text{x } 2.1} \end{array}$

10. $\begin{array}{r} 4.2 \\ \underline{\text{x } .602} \end{array}$

11. $35\overline{)70.35}$

12. $14\overline{)50.512}$

13. $23\overline{)74.888}$

Rename Fractions, Percents, and Decimals

Hints/Guide:

To convert fractions into decimals, we start with a fraction, such as $\frac{3}{5}$, and divide the numerator (the top number of the fraction) by the denominator (the bottom number of the fraction). So:

$5\overset{.6}{\overline{)3.0}}$ and the fraction $\frac{3}{5}$ is equivalent to the decimal 0.6

To convert a decimal to a percent, we multiply the decimal by 100 (percent means a ratio of a number compared to 100). A short-cut is sometimes used of moving the decimal point two places to the right (which is equivalent to multiplying a number by 100), so $0.6 \cdot 100 = 60$ and $\frac{3}{5} = 0.6 = 60\%$.

To convert a percent to a decimal, we divide the percent by 100,
60% is the same as 60 ÷ 100, which is 0.6, so 60% = 0.6

To convert a fraction into a percent, we can use proportions to solve, so
$\frac{3}{5} = \frac{x}{100}$ and using cross products to solve, 5x = 300 or x = 60%

Exercises: Complete the chart

	Fraction	**Decimal**	**Percent**
1.		0.04	
2.			125%
3.	$\frac{2}{3}$		
4.		1.7	
5.			0.6%
6.	$3\frac{1}{2}$		
7.		0.9	
8.			70%
9.	$\frac{17}{25}$		
10.		0.007	

Add and Subtract Mixed Numbers

Hints/Guide:

When adding mixed numbers, we can add the whole numbers and the fractions separately, then simplify the answer. For example:

$$4\frac{1}{3}+2\frac{3}{4}=4\frac{8}{24}+2\frac{18}{24}=6\frac{26}{24}=6+1\frac{2}{24}=7\frac{2}{24}=7\frac{1}{12}$$

When subtracting mixed numbers, we subtract the whole numbers and the fractions separately, then simplify the answer. For example:

$$7\frac{3}{4}-2\frac{15}{24}=7\frac{18}{24}-2\frac{15}{24}=5\frac{3}{24}=5\frac{1}{8}$$

$$5\frac{1}{4}-3\frac{3}{8}=5\frac{2}{8}-3\frac{3}{8}=4\frac{10}{8}-3\frac{3}{8}=1\frac{5}{8}$$ Note: regrouping needed in order to subtract

Exercises: Solve in lowest terms. No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. $3\frac{1}{2}+5\frac{3}{5}=$

2. $6\frac{17}{25}+8\frac{4}{7}=$

3. $6\frac{2}{3}+9\frac{7}{9}=$

4. $8\frac{3}{10}-6\frac{7}{9}=$

5. $9\frac{7}{15}-2\frac{7}{12}=$

6. $12\frac{8}{9}-7\frac{3}{4}=$

Multiply and Divide Mixed Numbers

Hints/Guide:

To multiply mixed numbers, we can first convert the mixed numbers into improper fractions. This is done my multiplying the denominator by the whole number part of the mixed number and them adding the numerator to this product. This sum is the numerator of the improper fraction. The denominator of the improper fraction is the same as the denominator of the mixed number.

For example: $3\frac{2}{5}$ leads to $3 \cdot 5 + 2 = 17$, so $3\frac{2}{5} = \frac{17}{5}$.

Once the mixed numbers are converted into improper fractions, we multiply and simplify just as with regular fractions. For example: $5\frac{1}{5} \bullet 3\frac{1}{2} = \frac{26}{2} \bullet \frac{7}{2} = \frac{182}{10} = 18\frac{2}{10} = 18\frac{1}{5}$

To divide mixed numbers, we must convert to improper fractions then multiply by the reciprocal of the second fraction and simplify. For example: $2\frac{1}{2} \div 3\frac{1}{3} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \bullet \frac{3}{10} = \frac{15}{20} = \frac{3}{4}$

Exercises: Solve in lowest terms. No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. $6\frac{2}{3} \bullet 7\frac{3}{7} =$

2. $3\frac{1}{3} \bullet 6\frac{4}{5} =$

3. $7\frac{1}{8} \bullet 6 =$

4. $4\frac{1}{4} \div \frac{5}{7} =$

5. $3\frac{2}{3} \div 4\frac{3}{7} =$

6. $\frac{3}{4} \div 2\frac{3}{11} =$

Laws of Exponents

Hints/Guide:

There are certain rules when dealing with exponents that we can use to simplify problems. They are:

Adding powers $a^m a^n = a^{m+n}$

Multiplying powers $(a^m)^n = a^{mn}$

Subtracting powers $\frac{a^m}{a^n} = a^{m-n}$

Negative powers $a^{-n} = \frac{1}{a^n}$

To the zero power $a^0 = 1$

Here are some examples of problems simplified using the above powers:

$4^3 \bullet 5^5 = 4^8$ $\quad (4^3)^3 = 4^9$ $\quad 4^5 \div 4^3 = 4^2$ $\quad 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$ $\quad 4^0 = 1$

Exercises: Simplify the following problems using exponents (Do not multiply out).

1. $5^2 \cdot 5^4 =$

2. $7^{-3}7^5 =$

3. $(12^4)^3 =$

4. $(6^5)^2 =$

5. $5^9 \div 5^4 =$

6. $10^3 \div 10^{-5} =$

7. $7^{-3} =$

8. $3^{-4} =$

9. $124^0 =$

10. $-9^0 =$

11. $(3^5 \bullet 3^2)^3 =$

12. $5^3 \bullet 5^4 \div 5^7 =$

Find Percent of a Number

Hints/Guide:

To determine the percent of a number, we must first convert the percent into a decimal by dividing by 100 (which can be short-cut by moving the decimal point in the percentage two places to the left), then multiplying the decimal by the number. For example:

$$4.5\% \text{ of } 240 = 4.5\% \cdot 240 = 0.045 \cdot 240 = 10.8$$

Exercises: Solve for n.

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. 30% of 40 = n
2. 7.5% of 42 = n
3. 150% of 320 = n
4. 15% of 54 = n
5. 0.65% of 320 = n
6. 80% of 9 = n
7. 9% of 7 = n
8. 150% of 38 = n
9. 215% of 348 = n
10. 70% of 30 = n

Solve Problems Using Percents

Hints/Guide:

When solving percent problems, we apple the rules for finding percent of a number in realistic situations. For example, to ding the amount of sales tax on a $450.00 item if the tax rate is 5%, we find 5% of 450 ($.05 \cdot 450 = 22.5$), and then label our answer in dollars, getting $22.50.

Exercises: Solve the following items.
SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. Susie has just bought a pair of jeans for $49.95, a sweater for $24.50, and a jacket for $85.95. The sales tax is 5%. What is her total bill?

2. Jack bought a set of golf clubs for $254.00 and received a rebate of 24%. How much was the rebate?

3. A construction manager calculates it will cost $2,894.50 for materials for her next project. She must add in 12.5% for scrap and extras. What will be the total cost?

4. The regular price for a video game system is $164.50 but is on sale for 30% off. What is the amount of the discount?

   What is the sale price?

5. Cindy earns a 15% commission on all sales. On Saturday, she sold $985.40 worth of merchandise. What was the amount of commission she earned on Saturday?

6. The band had a fundraiser and sold $25,800 worth of candy. They received 38% of this amount for themselves. How much did they receive?

Integers I

Hints/Guide:

To add integers with the same sign (both positive or both negative), add their absolute values and use the same sign. To add integers of opposite signs, find the difference of their absolute values and then take the sign of the larger absolute value.

To subtract integers, add its additive inverse. For example, 6 – 11 = 6 + -11 = -5

Exercises: Solve the following problems.

1. (-4) + (-5) =
2. –9 – (-2) =
3. 6 - (-9) =
4. (-6) – 7 =
5. 7 – (-9) =
6. 15 – 24 =
7. (-5) + (-8) =
8. –15 + 8 – 8 =
9. 14 + (-4) – 8 =
10. 14.5 – 29 =
11. –7 – 6.85 =
12. –8.4 – (-19.5) =
13. 29 - 16 + (-5) =
14. –15 + 8 – (-19.7) =
15. 45.6 – (-13.5) + (-14) =
16. –15.98 – 6.98 – 9 =
17. –7.24 + (-6.28) – 7.3 =
18. 29.45 – 56.009 – 78.2 =
19. 17.002 + (-7) – (-5.23) =
20. 45.9 – (-9.2) + 5 =

Solving Equations I

Hints/Guide:

The key in equation solving is to isolate the variable, to get the letter by itself. In one-step equations, we merely undo the operation - addition is the opposite of subtraction and multiplication is the opposite of division. Remember the golden rule of equation solving: If we do something to one side of the equation, we must do the exact same thing to the other side.

Examples:

1. $x + 5 = 6$
   $\underline{-5 \quad -5}$
   $x = 1$
   Check: $1 + 5 = 6$
   $6 = 6$

2. $t - 6 = 7$
   $\underline{+6 \quad +6}$
   $t = 13$
   Check: $13 - 6 = 7$
   $7 = 7$

3. $\frac{4x}{4} = \frac{16}{4}$
   $x = 4$
   Check: $4\,(4) = 16$
   $16 = 16$

4. $6 \cdot \frac{r}{6} = 12 \cdot 6$
   $r = 72$
   Check: $72 \div 6 = 12$
   $12 = 12$

Exercises: Solve the following problems: No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $x + 8 = -13$
2. $t - (-9) = 4$
3. $-4t = -12$
4. $\frac{r}{4} = 24$
5. $y - 4 = -3$
6. $h + 8 = -5$
7. $\frac{p}{8} = -16$
8. $-5k = 20$
9. $-9 - p = 17$

Inequalities

Hints/Guide:

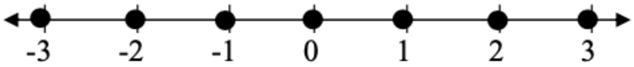
In solving inequalities, the solution process is very similar to solving equalities. The goal is still to isolate the variable, to get the letter by itself. However, the one difference between equations and inequalities is that when solving inequalities, when we multiply or divide by a negative number, we must change the direction of the inequality. Also, since an inequality as many solutions, we can represent the solution of an inequality by a set of numbers or by the numbers on a number line.

Inequality - a statement containing one of the following symbols:

< is less than
> is greater than
≤ is less than or equal to
≥ is greater than or equal to
_ is not equal to

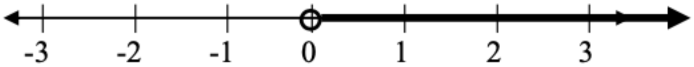
Examples:

1. Integers between -4 and 4.



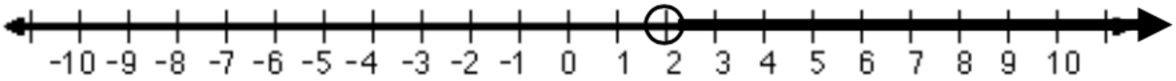
2. All numbers between -4 and 4.

3. The positive numbers.



So, to solve the inequality -4x < -8 becomes $\frac{-4x}{-4} < \frac{-8}{-4}$

and therefore x > 2 is the solution (this is because whenever we multiply or divide an inequality by a negative number, the direction of the inequality must change) and can be represented as:



Exercises: Solve the following problems: No Calculators!

1. 4x > 9

2. -5t ≥ -15

3. $\frac{x}{2} \geq 3$

4. $\frac{x}{-4} > 2$

Domain and Range - Ordered pairs

Sheet 1

A) Find the domain and range of each relation.

1) {(1, –1), (2, –3), (0, 5), (–1, 3), (4, –5), (–1, 5), (4, –4)}

Domain : ____________________

Range : ____________________

2) {(3, –2), (–8, –7), (0, 6), (–3, 4), (6, –3), (–1, 6), (5, –3)}

Domain : ____________________

Range : ____________________

3) {(10, –5), (–16, –8), (15, 9), (–4, 19), (6, –7)}

Domain : ____________________

Range : ____________________

4) {(5, –4), (7, –9), (0, 9), (–12 , 3), (9, 4), (–6, –3), (8, 2)}

Domain : ____________________

Range : ____________________

5) {(17, –9), (10, –5), (8, 3), (8, 4), (6, –14)}

Domain : ____________________

Range : ____________________

6) {(5, 5), (3, 8), (5, 4), (7, 5), (13, 8), (6, 2)}

Domain : ____________________

Range : ____________________

7) {(19, 12), (11, 5), (2, 2), (–4, 16), (6, 5), (–2, 1), (3, –3)}

Domain : ____________________

Range : ____________________

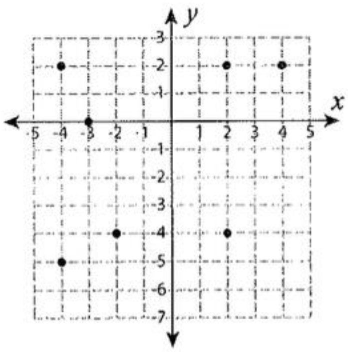
8) {(3, –2), (–3, –2), (1, 4), (–6, 5), (1, 3), (–20, 7)}

Domain : ____________________

Range : ____________________

B) Find the domain and range of ordered pairs represented on the graph.

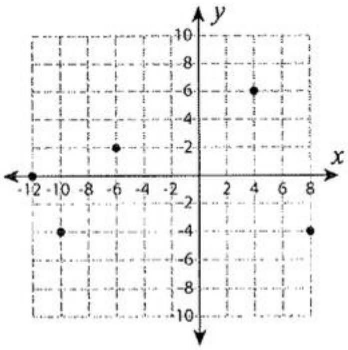
9)



Domain : ________________

Range : ________________

10)



Domain : ________________

Range : ________________

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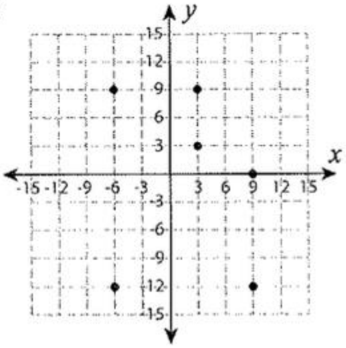


Domain : ________________

Range : ________________

Key Ideas

Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Numbers $2^3 \cdot 2^4 = 2^{3+4} = 2^7$ **Algebra** $a^m \cdot a^n = a^{m+n}$

Power of a Power Property

Words To find a power of a power, multiply the exponents.

Numbers $(3^5)^4 = 3^{5 \cdot 4} = 3^{20}$ **Algebra** $(a^m)^n = a^{m \cdot n} = a^{mn}$

EXAMPLE Multiplying Powers with the Same Base

$(-4)^3 \cdot (-4)^4 = (-4)^{3+4}$ Product of Powers Property

$= (-4)^7$ Simplify.

EXAMPLE Finding a Power of a Power

$(x^3)^4 = x^{3 \cdot 4}$ Power of a Power Property

$= x^{12}$ Simplify.

Write the product using exponents.

1. $(-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot x \cdot x$

2. $(-2) \cdot (-2) \cdot (-2) \cdot a \cdot a \cdot a \cdot a$

3. $2.4 \cdot 2.4 \cdot 2.4 \cdot 2.4 \cdot m \cdot m \cdot m$

4. $\left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot n \cdot n \cdot n \cdot n \cdot n$

Evaluate the expression.

5. -10^4

6. $(-10)^4$

7. $\left(\frac{1}{3}\right)^3$

8. $-\left(-\frac{1}{2}\right)^4$

9. 2.5^3

10. $(-2)^6$

11. -1.2^4

12. $-(-5)^3$

Simplify the expression. Write your answer as a power.

13. $5^3 \cdot 5^3$

14. $n^4 \cdot n^5$

15. $(-3)^6 \cdot (-3)^4$

16. $g^7 \cdot g^2$

17. $(x^4)^2$

18. $((-4.3)^3)^6$

19. $(a^{12})^3$

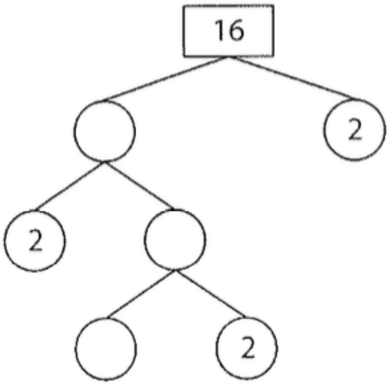
20. $(0.5^7)^6$

Prime Factor Tree

ES1

Complete the prime factor tree for each number.

1)



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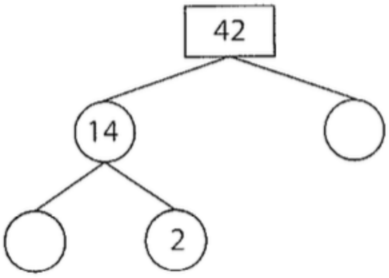

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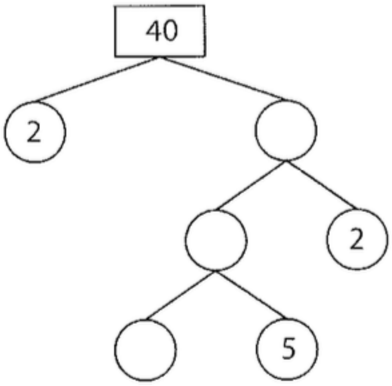

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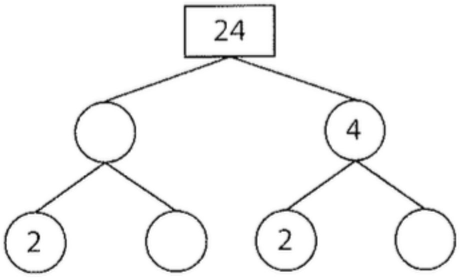

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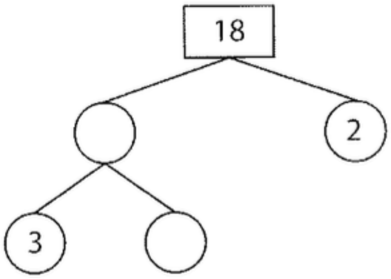

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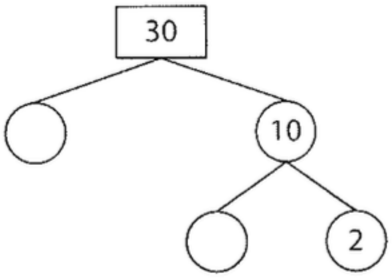
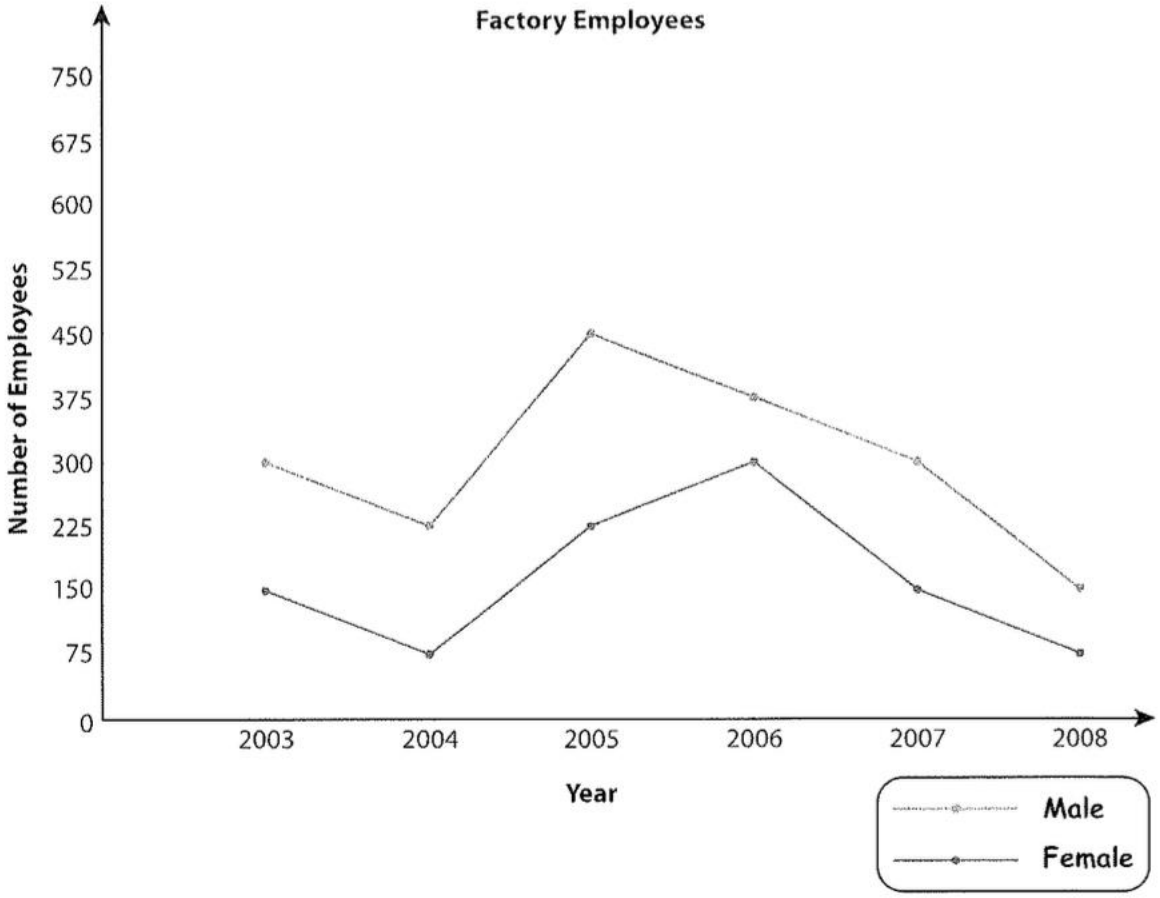


Double Line Graph - Factory Employees

A production company has both male and female staff working in the assembly unit. The graph shows the number of male and female employees in the company from 2003 to 2008. Read the graph and answer the questions.



1) Which year had the most number of female employees? ________________

2) How many employees worked in the year 2003? ________________

3) Which year had 225 male and female employees altogether? ________________

4) What is the difference on the number of male and female employees during the year 2004? ________________

5) Were the number of male employees more compared to the female employees in the year 2005? Yes or No. ________________

Properties of Operations

Commutative Property of Addition:
$a + b = b + a$

Commutative Property of Multiplication:
$a \times b = b \times\ a$

Associative Property of Addition:
$(a + b) + c = a + (b + c)$

Associative Property of Multiplication:
$(a \times b) \times c = a \times (b \times c)$

Identity Property of Addition:
$a +\ 0 = a$

Identity Property of Multiplication:
$a \times 1 = a$

Name the property illustrated by each expression.

1. $8 \times 12 = 12 \times 8$
2. $3 \times (2 \times 5) = (3 \times 2) \times 5$
3. $2 + 5 + 12 = 5 + 2 + 12$
4. $xy + 0 = xy$
5. $1x = x$
6. $5 + 7 = 7 + 5$
7. $3 + (4 + 5) = 3 + (5 + 4)$
8. $3xy = 3xy(1)$
9. $(4 + 8) + 5 = 4 + (8 + 5)$
10. $5 \times 6 \times 8 = 8 \times 5 \times 6$

Evaluating Expressions and Formulas

To evaluate an expression, first replace the variable by a given value. Then simplify the resulting numerical expression.

Evaluate the expression when $x = -2$ and $y = 5$.

1. $x + y$
2. $x^2 + y^3$
3. $2x - y$
4. $-2(y - 2x)$
5. $\frac{3x-y}{11}$
6. $\frac{x}{3-y}$

Order of Operations

When several operations are indicated in a numerical expression, proceed in the following order: work within the parentheses, expand each power, multiply and divide (whichever comes first), and finally, add or subtract (whichever comes first).

PEMDAS (“**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally”) is an acronym that provides a good way to remember your order of operation.

P: Parentheses
E: Exponents
MD: Multiply or Divide, whichever comes first
AS: Add or Subtract, whichever comes first

Simplify.

1. $2^4 - 3(3^2 - 8)$

2. $(4^2 + 10)4 - 10(5^2 - 20)$

3. $4^2 - 4(5^2 - 32 \div 8 \cdot 4)$

4. $(8 \cdot 5 \div 10 + 2)(2^5 - 8^2 \div 2)$

5. $5^2 - 3\big[6 + (-2)\big(20 + (-15)\big)\big]$

6. $[4^3 + (-10)(30 - 8 \cdot 5)]$

7. $[15 - 3(4^2 - 10) + 25 \div 5 \cdot 15]$

8. $\{10 - 5[20 - 2(3^2 + 1)]\}$

9. $|-32| + 32$

10. $\frac{48-24\div 2^3}{3+2\cdot 6}$

Central Tendencies
(Mean, Median, Mode, and Range)

Mean is the sum of the values in a set of data divided by the number of values.

Median is the middle value of a set of data written in ascending order. If there are two middle values, the median is the mean of those values.

Mode is the most frequent value in a set of data.

Range is the difference between the greatest and least value in a set of data.

Exercises:

Find the mean, median, mode, and range of each set of data.

1. 108, 93, 426, 766, 518, 210

2. 21.5, 35.5, 49.5, 16.3, 35.5